## Lesson 3

Objective: Use exponents to name place value units and explain patterns in the placement of the decimal point.

## Suggested Lesson Structure

| Fluency Practice | (15 minutes) |
| :--- | :--- |
| Application Problems | (7 minutes) |
| Concept Development | (28 minutes) |
| Student Debrief | (10 minutes) |
| Total Time | (60 minutes) |

## Fluency Practice (15 minutes)

- Multiply by 3 3.OA. 7
- State the Unit as a Decimal 5.NBT. 2
- Multiply by 10, 100, and 1000 5.NBT. 2

Total Time
(8 minutes)

(4 minutes)
(3 minutes)

## Sprint: Multiply by 3 (8 minutes)

Materials: (S) Multiply by 3 Sprint.
Notes: This fluency will review foundational skills learned in Grades 3 and 4.

## State the Unit as a Decimal—Choral Response (4 minutes)

Notes: Reviewing these skills will help students work towards mastery of decimal place value, which will help them apply their place value skills to more difficult concepts.

T: (Write 9 tenths = $\qquad$ .)
S: 0.9
T: (Write 10 tenths = $\qquad$ .)
S: 1.0
T: Write 11 tenths = $\qquad$ .)
S: 1.1
T: (Write 12 tenths = $\qquad$ .)

S: 1.2
T: (Write 18 tenths = $\qquad$ .)

S: 1.8
T: (Write 28 tenths = $\qquad$ _.)

S: 2.8
T: (Write 58 tenths = $\qquad$ .)
S: 5.8
Repeat the process for 9 hundredths, 10 hundredths, 20 hundredths, 60 hundredths, 65 hundredths, 87 hundredths, and 118 tenths. (This last item is an extension.)

## Multiply and Divide by 10, 100, and 1000 (3 minutes)

Materials: (S) Personal white boards.
Notes: This fluency drill will review concepts taught in Lesson 2.
T: (Project place value chart from millions to thousandths.) Write two 1 thousandths disks and the number below it.
S: (Students write two 1 thousandths disks in the thousandths column. Below it, they write 0.002.)
T: Multiply by 10. Cross out each disk and the number 2 to show that you're changing its value.
S: (Students cross out each 1 thousandths disk and the 2. They draw arrows to the hundredths column and write two 1 hundredth disks. Below it, they write 2 in the hundredths column and 0 in the ones and tenths column.)

Repeat the process for the possible sequence: $0.004 \times 100$; $0.004 \times 1000 ; 1.004 \times 1000 ; 1.024 \times 100 ; 1.324 \times 100 ; 1.324 \times$ 10 ; and $1.324 \times 1000$.

Repeat the process for dividing by 10, 100, and 1000 for this possible sequence: $4 \div 10 ; 4.1 \div 10 ; 4.1 \div 100 ; 41 \div 1000$; and $123 \div 1000$.

## Application Problem (7 minutes)

Jack and Kevin are creating a mosaic by using fragments of broken tiles for art class. They want the mosaic to have 100 sections. If each section requires 31.5 tiles, how many tiles will they need to complete the mosaic? Explain your reasoning with a place value chart.

## NOTES ON <br> MULTIPLE MEANS OF ACTION AND EXPRESSION:

Very large numbers like one million and beyond easily capture the imagination of students. Consider allowing students to research and present to classmates the origin of number names like googol and googleplex. Connections to literacy can also be made with books about large numbers, such as How Much is a Million by Steven Kellogg, A Million Dots by Andrew Clements, Big Numbers and Pictures That Show Just How Big They Are by Edward Packard and Sal Murdocca.
The following benchmarks may help students appreciate just how large a googol is.

- There are approximately $10^{24}$ stars in the observable universe.
- There are approximately $10^{80}$ atoms in the observable universe.
- A stack of 70 numbered cards can be ordered in approximately 1 googol different ways. That means that that the number of ways a stack of only 70 cards can be shuffled is more than the number of atoms in the observable universe.


## Concept Development (28 minutes)

Materials: (S) Personal white boards

## Problem 1

$\mathrm{T}: \quad$ (Draw or project chart, adding numerals as discussion unfolds.)

|  |  |  |  | 100 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  | $10 \times 10$ | $10 \times 1$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

T: (Write $10 \times$ $\qquad$ $=10$ on the board.) On your personal board, fill in the missing factor to complete this number sentence.
S: (Students write.)
T: (Write $10 \times$ $\qquad$ $=100$ on the board.) Fill in the missing factor to complete this number sentence.

S: (Students write.)
T: This time, using only 10 as a factor, how could you multiply to get a product of 1000 ? Write the multiplication sentence on your personal board.

S: $\quad 10 \times 10 \times 10=1000$.
T: Work with your partner. What would the multiplication sentence be for 10,000 using only 10 as a factor? Write on your personal board.
S: (Students write.)
T: How many factors of 10 did we have to multiply to get to 1000 ?
S: 3.
T: How many factors of 10 do we have to multiply to get 10,000 ?
S: 4.
T: Say the number sentence.
S: $\quad 10 \times 10 \times 10 \times 10=10,000$.
T: How many zeros are in our product, 10,000?
S: 4 zeros.
T: What patterns do you notice? Turn and share with your partner.
S : The number of zeros is the same on both side of the equation. $\rightarrow$ The number of zeros in the product is the same as the number of zeros in the factors. $\rightarrow$ I see three zeros on the left side, and there are three zeros on the right side for $10 \times 10 \times 10=1000 . \rightarrow$ The 1 moves one place to the left every time we multiply by $10 . \rightarrow$ It's like a place value chart. Each number is 10 times as much as the last one.

T: Using this pattern, how many factors of 10 do we have to multiply to get 1 million? Work with your partner to write the multiplication sentence.

S: (Students write.)
T: How many factors of 10 did you use?
S: 6
T: Why did we need 6 factors of 10 ?
S: 1 million has 6 zeros.
T: We can use an exponent (write term on the board) to represent how many times we use 10 as a factor. We can write $10 \times 10$ as $10^{2}$. (Add to the chart.) We say, "Ten to the second power." The 2 (point to exponent) is the exponent and it tells us how many times to use 10 as a factor.
T: How do you express 1000 using exponents? Turn and share with your partner.
S: We multiply $10 \times 10 \times 10$, that's three times, so the answer is $10^{3} . \rightarrow$ There are three zeros in 1000 , so it's ten to the third power.
T : Working with your partner, complete the chart using the exponents to represent the each value on the place value chart.

| $1,000,000$ | 100,000 | 10,000 | 1000 | 100 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $(10 \times 10 \times 10) \times(10 \times 10 \times 10)$ | $10 \times 10 \times(10 \times 10 \times 10)$ | $10 \times(10 \times 10 \times 10)$ | $(10 \times 10 \times 10)$ | $10 \times 10$ | $10 \times 1$ |
| $10^{6}$ | $10^{5}$ | $10^{4}$ | $10^{3}$ | $10^{2}$ | $10^{1}$ |

After reviewing the chart with the students, challenge them to multiply 10 one hundred times. As some start to write it out, others may write $10^{100}$, a googol, with exponents.

T: Now look at the place value chart; let's read our powers of 10 and the equivalent values.
S: Ten to the second power equals 100; ten to the third power equals 1000. (Continue to read chorally up to 1 million.)
T: Since a googol has 100 zeros, write it using an exponent on your personal board.
S: (Students write $10^{100}$.)

## Problem 2

$10^{5}$
T : Write ten to the fifth power as a product of tens.
S: $\quad 10^{5}=10 \times 10 \times 10 \times 10 \times 10$.

## NOTES ON <br> MULTIPLE MEANS OF REPRESENTATIONS:

Providing non-examples is a powerful way to clear up mathematical misconceptions and generate conversation around the work. Highlight those examples such as $10^{5}$ pointing out its equality to $10 \times 10 \times 10$ $\times 10 \times 10$ but not to $10 \times 5$ or even $5^{10}$.
Allowing students to explore a calculator and highlighting the functions used to calculate these expressions (e.g., $10^{\wedge} 5$ versus $10 \times 5$ ) can be valuable.

T : Find the product.
S: $\quad 10^{5}=100,000$.
Repeat with more examples as needed.

## Problem 3

$10 \times 100$
T: Work with your partner to write this expression using an exponent on your personal board. Explain your reasoning.
S: I multiply $10 \times 100$ to get 1000 , so the answer is ten to the third power. $\rightarrow$ There are 3 factors of 10. $\rightarrow$ There are three 10's. I can see one 10 in the first factor and 2 more tens in the second factor.

Repeat with $100 \times 1000$ and other examples as needed.

## Problems 4-5

$3 \times 10^{2}$
$3.4 \times 10^{3}$
T: Compare this expression to the ones we've already talked about.
S: These have factors other than 10.
T: Write $3 \times 10^{2}$ without using an exponent. Write it on your personal board.
S: $3 \times 100$.
T: What's the product?
S: 300.
T: If you know that $3 \times 100$ is 300 , then what is $3 \times 10^{2}$ ? Turn and explain to your partner.
S: The product is also 300. $10^{2}$ and 100 are same amount so the product will be the same.
T: Use what you learned about multiplying decimals by 10, 100, and 100 and your new knowledge about exponents to solve $3.4 \times 10^{3}$ with your partner.
S: (Students work.)
Repeat with $4.021 \times 10^{2}$ and other examples as needed.
Have students share their solutions and reasoning about multiplying decimal factors by powers of ten. In particular, students should articulate the relationship between the exponent and how the values of the digits change and placement of the decimal in the product.

## Problems 6-7

$700 \div 10^{2}$
$7.1 \div 10^{2}$
T: Write $700 \div 10^{2}$ without using an exponent and find the quotient. Write it on your personal board.
S: $\quad 700 \div 100=7$
T: If you know that $700 \div 100$ is 7 , then what is $700 \div 10^{2}$ ? Turn and explain to your partner.
S: The quotient is 7 because $10^{2}=100$.
T: Use what you know about dividing decimals by multiples of 10 and your new knowledge about exponents to solve $7.1 \div 10^{2}$ with your partner.
S: (Students work.)

T: Tell your partner what you notice about the relationship between the exponents and how the values of the digits change. Also discuss how you decided where to place the decimal.

Repeat with more examples as needed.

## Problems 8-9

Complete this pattern: $0.043 \quad 4.3 \quad 430$
T: (Write the pattern on the board.) Turn and talk with your partner about the pattern on the board. How is the value of the 4 changing as we move to the next term in the sequence? Draw a place value chart to explain your ideas as you complete the pattern and use an exponent to express the relationships.
S: The 4 moved two places to the left. $\rightarrow$ Each number is being multiplied by 100 to get the next one. $\rightarrow$ Each number is multiplied by 10 twice. $\rightarrow$ Each number is multiplied by $10^{2}$.

Repeat with 6,300,000; $\qquad$ ; 630; 6.3; $\qquad$ and other patterns as needed.

T: As you work on the Problem Set, be sure you are thinking about the patterns that we've discovered today.

## Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

## Student Debrief (10 minutes)

Lesson Objective: Use exponents to name place value units and explain patterns in the placement of the decimal point.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.


- What is an exponent and how can exponents be useful in representing numbers? (This question could also serve as a prompt for math journals. Journaling about new vocabulary throughout the year can be a powerful way for students to solidify their understanding of new terms.)
- How would you write 1000 using exponents? How would you write it as a multiplication sentence using only 10 as a factor?
- Explain to your partner the relationship we saw between the exponents and the number of the places the digit shifted when you multiply or divide by a power of 10 .
- How are the patterns you discovered in Problem 3 and 4 in the Problem Set alike?

Give students plenty of opportunity to discuss the error patterns in Problem 6(a) and 6(b). These are the most common misconceptions students hold when dealing with exponents, so it is worth the time to see that they

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    c) 94,300 9.40 943 9.3 243 0.943
```



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    e) 0.075 7.5 750 75,000 1,500,000 750,000000
        7) Explain how you found the missing numbers in set (b). Be sure to inctude your reasoning about
        the number of zeros in your numbers and how you placed me decima..
        I noticed that the next number in the pattern had two zomus
    less than the number before, so the numbers were being divided
    by }1\mp@subsup{0}{}{2}\mathrm{ . That means the numbe sh.its two places to the right.
    0) Explain how you found the missing numbers in set (e). Be sure to include your reasoning about
    the number of ferosin in your numbers and how you placed the decimal.
    I noticed that the whole numbers added two zeros, so the
numbers were }1\mp@subsup{0}{}{2}\mathrm{ greater from the previous number. That helped me know
to shift 7.5 two places to the right and get 0.075.
    6. Shaunnie and Marton missed the lesson on exponents., You notice thar Shaunmie incorrectly wrote
        10}=50\mathrm{ on her paper, and you notice that Marton incorrectly wrote 2.5 * 10% =2.500 on his paper.
        a. What mistake has shaunnie made? Explain to her using words, numbers and pictures why
        her thinking is incorrect and what she needs to do to correct her answer:
        Shewnie thinks that }1\mp@subsup{0}{}{5}\mathrm{ is 10 }15.1\mp@subsup{0}{}{5}=10\times10\times10\times10\times10
        10}=100.000\mathrm{ .
            b. What mistake has Marton made? Explain to Marlon using words, numbers and pictures why
        .What mistake has Marton made? Explain to Marlon using words, number
            Marlon made the mistuke of just adding zeros at the
    end of 2.5. He needs to remember that multiplying by }1\mp@subsup{0}{}{2
    Makes a number l00 times greater, which 
L. COMMON Unmen encracie}\mp@subsup{}{}{\mathrm{ ny }
``` do not become firmly held.

\section*{Exit Ticket (3 minutes)}

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{4}{|l|}{A} & \multirow[t]{2}{*}{\# Correct} \\
\hline & & & & \\
\hline 1 & \(1 \times 3=\) & 23 & \(10 \times 3=\) & \\
\hline 2 & \(3 \times 1=\) & 24 & \(9 \times 3=\) & \\
\hline 3 & \(2 \times 3=\) & 25 & \(4 \times 3=\) & \\
\hline 4 & \(3 \times 2=\) & 26 & \(8 \times 3=\) & \\
\hline 5 & \(3 \times 3=\) & 27 & \(5 \times 3=\) & \\
\hline 6 & \(4 \times 3=\) & 28 & \(7 \times 3=\) & \\
\hline 7 & \(3 \times 4=\) & 29 & \(6 \times 3=\) & \\
\hline 8 & \(5 \times 3=\) & 30 & \(3 \times 10=\) & \\
\hline 9 & \(3 \times 5=\) & 31 & \(3 \times 5=\) & \\
\hline 10 & \(6 \times 3=\) & 32 & \(3 \times 6=\) & \\
\hline 11 & \(3 \times 6=\) & 33 & \(3 \times 1=\) & \\
\hline 12 & \(7 \times 3=\) & 34 & \(3 \times 9=\) & \\
\hline 13 & \(3 \times 7=\) & 35 & \(3 \times 4=\) & \\
\hline 14 & \(8 \times 3=\) & 36 & \(3 \times 3=\) & \\
\hline 15 & \(3 \times 8=\) & 37 & \(3 \times 2=\) & \\
\hline 16 & \(9 \times 3=\) & 38 & \(3 \times 7=\) & \\
\hline 17 & \(3 \times 9=\) & 39 & \(3 \times 8=\) & \\
\hline 18 & \(10 \times 3=\) & 40 & \(11 \times 3=\) & \\
\hline 19 & \(3 \times 10=\) & 41 & \(3 \times 11=\) & \\
\hline 20 & \(3 \times 3=\) & 42 & \(12 \times 3=\) & \\
\hline 21 & \(1 \times 3=\) & 43 & \(3 \times 13=\) & \\
\hline 22 & \(2 \times 3=\) & 44 & \(13 \times 3=\) & \\
\hline
\end{tabular}
(C) Bill Davidson
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{B} & \multicolumn{2}{|l|}{Improvement} & \# Correct \\
\hline 1 & \(3 \times 1=\) & 23 & \(9 \times 3=\) & \\
\hline 2 & \(1 \times 3=\) & 24 & \(3 \times 3=\) & \\
\hline 3 & \(3 \times 2=\) & 25 & \(8 \times 3=\) & \\
\hline 4 & \(2 \times 3=\) & 26 & \(4 \times 3=\) & \\
\hline 5 & \(3 \times 3=\) & 27 & \(7 \times 3=\) & \\
\hline 6 & \(3 \times 4=\) & 28 & \(5 \times 3=\) & \\
\hline 7 & \(4 \times 3=\) & 29 & \(6 \times 3=\) & \\
\hline 8 & \(3 \times 5=\) & 30 & \(3 \times 5=\) & \\
\hline 9 & \(5 \times 3=\) & 31 & \(3 \times 10=\) & \\
\hline 10 & \(3 \times 6=\) & 32 & \(3 \times 1=\) & \\
\hline 11 & \(6 \times 3=\) & 33 & \(3 \times 6=\) & \\
\hline 12 & \(3 \times 7=\) & 34 & \(3 \times 4=\) & \\
\hline 13 & \(7 \times 3=\) & 35 & \(3 \times 9=\) & \\
\hline 14 & \(3 \times 8=\) & 36 & \(3 \times 2=\) & \\
\hline 15 & \(8 \times 3=\) & 37 & \(3 \times 7=\) & \\
\hline 16 & \(3 \times 9=\) & 38 & \(3 \times 3=\) & \\
\hline 17 & \(9 \times 3=\) & 39 & \(3 \times 8=\) & \\
\hline 18 & \(3 \times 10=\) & 40 & \(11 \times 3=\) & \\
\hline 19 & \(10 \times 3=\) & 41 & \(3 \times 11=\) & \\
\hline 20 & \(1 \times 3=\) & 42 & \(13 \times 3=\) & \\
\hline 21 & \(10 \times 3=\) & 43 & \(3 \times 13=\) & \\
\hline 22 & \(2 \times 3=\) & 44 & \(12 \times 3=\) & \\
\hline
\end{tabular}
(C) Bill Davidson

Name \(\qquad\) Date \(\qquad\)
1. Write the following in exponential form (e.g., \(100=10^{2}\) ).
a. \(10,000=\) \(\qquad\)
d. \(100 \times 100=\) \(\qquad\)
b. \(1000=\) \(\qquad\)
e. \(1,000,000=\) \(\qquad\)
c. \(10 \times 10=\) \(\qquad\) f. \(1000 \times 1000=\) \(\qquad\)
2. Write the following in standard form (e.g., \(5 \times 10^{2}=500\) ).
a. \(9 \times 10^{3}=\) \(\qquad\)
e. \(4.025 \times 10^{3}=\) \(\qquad\)
b. \(39 \times 10^{4}=\) \(\qquad\)
f. \(\quad 40.25 \times 10^{4}=\) \(\qquad\)
c. \(7200 \div 10^{2}=\) \(\qquad\)
g. \(725 \div 10^{3}=\) \(\qquad\)
d. \(7,200,000 \div 10^{3}=\) \(\qquad\)
h. \(7.2 \div 10^{2}=\) \(\qquad\)
3. Think about the answers to Problem 2(a-d). Explain the pattern used to find an answer when you multiply or divide a whole number by a power of 10.
4. Think about the answers to Problem 2(e-h). Explain the pattern used to place the decimal in the answer when you multiply or divide a decimal by a power of 10 .
5. Complete the patterns.
a. \(0.03 \quad 0.3 \quad 30\)
b. \(6,500,000\)

65,000 \(\qquad\) 6.5 \(\qquad\)
c. \(\qquad\) 9,430 \(\underline{\square}\)
94.3
9.43
\(\qquad\)
d. 9

9990 99,900 \(\qquad\)
\(\qquad\)
\(\qquad\)
e. \(\qquad\) \(7.5 \quad 750 \quad 75,000\) \(\qquad\)
\(\qquad\)
f. Explain how you found the missing numbers in set (b). Be sure to include your reasoning about the number of zeros in your numbers and how you placed the decimal.
g. Explain how you found the missing numbers in set (d). Be sure to include your reasoning about the number of zeros in your numbers and how you placed the decimal.
6. Shaunnie and Marlon missed the lesson on exponents. Shaunnie incorrectly wrote \(10^{5}=50\) on her paper, and Marlon incorrectly wrote \(2.5 \times 10^{2}=2.500\) on his paper.
a. What mistake has Shaunnie made? Explain using words, numbers, and pictures why her thinking is incorrect and what she needs to do to correct her answer.
b. What mistake has Marlon made? Explain using words, numbers, and pictures why his thinking is incorrect and what he needs to do to correct his answer.

Name \(\qquad\) Date \(\qquad\)
1. Write the following in exponential form and as a multiplication sentence using only 10 as a factor (e.g., \(100=10^{2}=10 \times 10\) ).
a. 1,000
\(=\) \(\qquad\) \(=\) \(\qquad\)
b. \(100 \times 100\)
\(=\) \(\qquad\) \(=\) \(\qquad\)
2. Write the following in standard form (e.g., \(4 \times 10^{2}=400\) ).
a. \(3 \times 10^{2}=\) \(\qquad\) c. \(800 \div 10^{2}=\) \(\qquad\)
b. \(\quad 2.16 \times 10^{4}=\) \(\qquad\)
d. \(754.2 \div 10^{3}=\) \(\qquad\)

Name \(\qquad\) Date \(\qquad\)
1. Write the following in exponential form (e.g., \(100=10^{2}\) ).
a. \(1000=\) \(\qquad\)
d. \(100 \times 10=\) \(\qquad\)
b. \(10 \times 10=\) \(\qquad\)
e. \(1,000,000=\) \(\qquad\)
c. \(100,000=\) \(\qquad\)
f. \(10,000 \times 10=\) \(\qquad\)
2. Write the following in standard form (e.g., \(4 \times 10^{2}=400\) ).
a. \(4 \times 10^{3}=\) \(\qquad\)
e. \(6.072 \times 10^{3}=\) \(\qquad\)
b. \(64 \times 10^{4}=\) \(\qquad\)
f. \(\quad 60.72 \times 10^{4}=\) \(\qquad\)
c. \(5300 \div 10^{2}=\) \(\qquad\)
g. \(948 \div 10^{3}=\) \(\qquad\)
d. \(5,300,000 \div 10^{3}=\) \(\qquad\) h. \(9.4 \div 10^{2}=\) \(\qquad\)
3. Complete the patterns.
a. \(0.02 \quad 0.2\) \(\qquad\) 20 \(\qquad\)
b. \(3,400,000\)
34,000 \(\qquad\) 3.4 \(\qquad\)
c. \(\qquad\) 8,570 \(\qquad\) \(85.7 \quad 8.57\) \(\qquad\)
d. \(4444440 \quad 44,400\) \(\qquad\)
\(\qquad\)
\(\qquad\)
e. \(\qquad\) 9.5950
95,000 \(\qquad\)
\(\qquad\)
4. After a lesson on exponents, Tia went home and said to her mom, "I learned that \(10^{4}\) is the same as 40,000 ." She has made a mistake in her thinking. Use words, numbers or a place value chart to help Tia correct her mistake.
5. Solve \(247 \div 10^{2}\) and \(247 \times 10^{2}\).
a. What is different about the two answers? Use words, numbers or pictures to explain how the decimal point shifts.
b. Based on the answers from the pair of expressions above, solve \(247 \div 10^{3}\) and \(247 \times 10^{3}\).```

