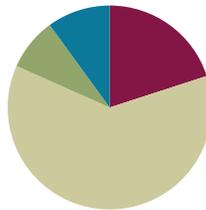## Lesson 6

Objective: Use varied protractors to distinguish angle measure from length measurement.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (37 minutes)
■ Student Debrief   (6 minutes)
  Total Time   **(60 minutes)**

Fluency Practice (12 minutes)

▪ Divide Using the Area Model **4.NBT.6**   (4 minutes)
▪ Draw and Identify Two-Dimensional Figures **4.G.1**   (4 minutes)
▪ Physiometry **4.G.1**   (4 minutes)
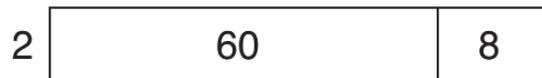
Divide Using the Area Model (4 minutes)

Materials: (S) Personal white boards

Note: This drill reviews G4–M3–Lesson 20 content.

 T: (Project area model that shows 68 ÷ 2.) Write a division expression for this area model.
 S: (Write 68 ÷ 2.)
 T: Label the length of each rectangle in the area model.
 S: (Write 30 above the 60 and 4 above the 8.)
 T: Solve using the standard algorithm.

2	60	8

Students do so.

Continue with the following possible suggestions: 69 ÷ 3, 78 ÷ 3, and 76 ÷ 4.

Draw and Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white boards, straightedge

Note: This fluency reviews terms introduced in G4–M4–Lessons 1–5.

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T: (Project $\overline{AB}$. Point to the A.) Say the term for what I'm pointing to?

S: Point A.

T: (Point to the B.) Say the term.

S: Point B.

T: (Point to $\overline{AB}$.) Say the term.

S: Line segment AB.

T: Use your rulers to construct $\overline{CD}$ on your boards.

S: (Draw $\overline{CD}$.)

T: Beneath $\overline{CD}$, draw $\overleftrightarrow{EF}$ that is parallel to $\overline{CD}$.

S: (Beneath $\overline{CD}$, draw $\overleftrightarrow{EF}$ that is parallel to $\overline{CD}$.)

T: Draw $\overrightarrow{GH}$ that begins on $\overleftrightarrow{EF}$ and runs perpendicular through $\overline{CD}$.

S: (Draw $\overrightarrow{GH}$ that begins on $\overleftrightarrow{EF}$ and runs perpendicular through $\overline{CD}$.)

T: What's the relationship between $\overrightarrow{GH}$ and $\overline{CD}$?

S: $\overrightarrow{GH}$ is perpendicular to $\overline{CD}$.

T: Draw $\overline{IJ}$ that is perpendicular to $\overline{KL}$.

S: (Draw $\overline{IJ}$. Draw $\overline{KL}$ that is perpendicular to $\overline{IJ}$.)

T: Draw $\overline{MN}$ that is perpendicular to $\overline{IJ}$ and parallel to $\overline{KL}$.

S: (Draw $\overline{MN}$ that is perpendicular to $\overline{IJ}$ and parallel to $\overline{KL}$.)

T: (Project a right angle ACB.) Name the angle.

S: $\angle ACB$.

T: What type of angle is it?

S: Right angle.

T: What's the relationship of $\overrightarrow{CA}$ and $\overrightarrow{CB}$?

S: They're perpendicular.

T: How many degrees are in $\angle ACB$?

S: 90 degrees.

T: (Project an acute angle DFE.) Name the angle.

S: $\angle DFE$.

T: (Beneath $\angle DFE$, write 30° or 150°.) Estimate. Is the measure of $\angle DFE$ 30° or 150°?

S: 30°.

T: How do you know?

S: Acute angles are less than 90°.

Continue with the other given angles.

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Physiometry (4 minutes)

Note: Kinesthetic memory is strong memory. This fluency reviews terms from G4–M4–Lessons 1–5.

T: Stand up.

S: (Stand up.)

T: Show me a right angle.

S: (Stretch one arm up directly at the ceiling. Stretch another arm directly towards a wall, parallel to the floor.)

T: Show me a different right angle.

S: (Stretch the arm pointing towards a wall directly up towards the ceiling. Move the arm pointing towards the ceiling so that it points directly towards the opposite wall.)

T: Show me an obtuse angle.

S: (Make an obtuse angle with arms.)

T: Show me an acute angle.

S: (Make an acute angle with arms.)

T: Make a right angle.

S: (Make a right angle with arms.)

T: Make an angle that measures approximately 30°.

S: (Move arms closer together, lessening the space between their arms, so that it is approximately 30°.)

T: Make an angle that measures approximately 60°.

S: (Open arms further apart to approximately 60°.)

Continue with the following possible sequence: 90°, 120°, 150°, 50°, 170°, 70°, and 180°.

T: What is the term for a 180° angle?

S: Line.

T: Make a line segment.

S: (Close fists.)

T: (Point at the classroom's back wall.) Point to the walls that run perpendicular to the wall I'm pointing to.

S: (Point to the side walls.)

T: (Point to the front wall.)

S: (Point to the side walls.)

Continue pointing to one side wall, the back wall, the other side wall, and the front wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I'm pointing to.

S: (Point to the front wall.)

Continue pointing to one side wall, the front wall, and the other side wall.

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Application Problem (5 minutes)

Materials: (S) 2 circles of different sizes (different colors if possible)

Fold Circle A and Circle B as you would to make a right angle template. Trace the folded perpendicular lines. How many right angles do you see at the center of each circle? Did the size of the circle matter?

Note: This Application Problem connects to G4–M4–Lesson 5 in which students found four right angles are within a circle. Students will find the number of right angles around the center point of different size circles as an introduction to arc length measure having no effect on angle measurement in this Concept Development.
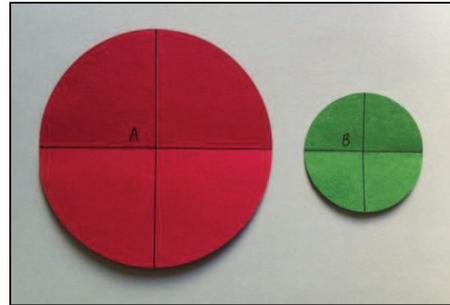
Concept Development (37 minutes)

Materials: (T) 2 circle cutouts from Application Problem, 2 pieces of wire the same length as the circumference of each circle cutout, Practice Sheet, straightedge, various protractors (S) 2 circle cutouts from Application Problem, Practice Sheet, straightedge, an assortment of protractors including at least one circular protractor and one 180° protractor.

Note: Providing a variety of protractors will allow students to distinguish angle measure from length measure. Students may share protractors during this activity. It is not necessary for every student to have two or three varied protractors of their own.

> **NOTES ON
> MULTIPLE MEANS OF
> REPRESENTATION:**
>
> Check that English language learners and others understand the meaning of the new math term *arc*. If necessary and possible, offer explanations in students' first language. Link *arc* to more familiar words or phrases such as *golden arches.*

**Problem 1: Explore the effect of angle size on arc length.
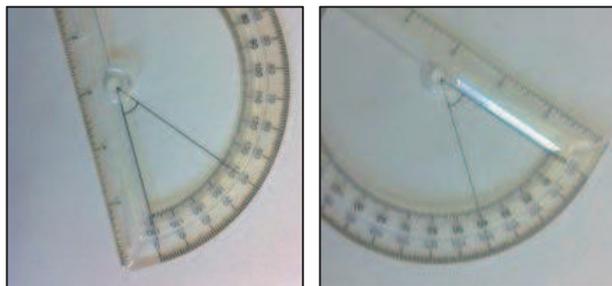Distinguish between angle and length measurement.**

- T: How many degrees are in a right angle?
- S: 90 degrees.
- T: Draw an arc on Circle A and Circle B (as pictured to the right).
- T: Trace your finger along each arc. Which circle has a longer arc?
- S   Circle A!
- T: But don't both arcs measure 90 degrees? Why are the arcs different lengths?
- S: I don't know. → Circle A is bigger, so maybe it needs a bigger arc.
- T: How many total degrees in this circle? (Point to Circle A.)

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S: 360 degrees.

T: How many total degrees in this circle? (Point to Circle B.)

S: 360 degrees.

T: So if I divide Circle A into 360 degrees, each arc length will be a little longer than the arc lengths in Circle B. I'm still measuring a quarter turn in each circle, and each arc is one fourth of the total distance around the circle.

T: Think of it also like taking the arc lengths from each circle and stretching them out into a line. (Model two wires that wrap the circumference of each circle stretched out in a line.) I can chop each wire into 360 equal-size pieces. Which arc will have smaller pieces?

S: The arc from Circle B.

T: Right! 90 degrees is one quarter of 360 degrees. (Cut each wire into four equal parts. Show one part from each wire is the same length as the arc of each circle.) Which arc is longer?

S: Circle A has a longer arc.

T: So does the length of the arc determine the measure of a given angle? Discuss this with your partner.

S: No! The arcs might be longer or shorter, but they could be measuring the same size angle. → No matter where the arc is, I just have to remember that arc is part of 360 degrees. → Right, because I could have a super tiny circle or a really big circle, but still the right angles measure 90 degrees.

T: Place Circle B on top of Circle A to show the length of the arc does not determine the degree measure.
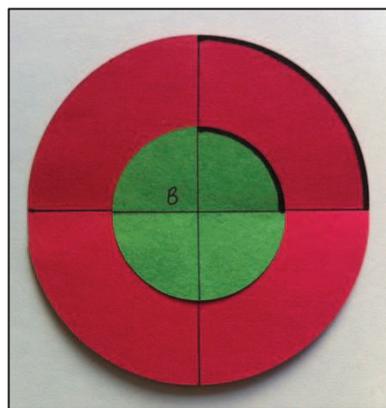
Problem 2: Use a 180° protractor to verify angle measure.

T: (Project ∠C and ∠D from the Practice Sheet.) What type of angle do you see?

S: Acute!

T: Discuss what you notice about the arc length in each angle.

S   The arc length in ∠C is longer than the one in ∠D. → The arcs are different lengths, but the angles look like they might be the same. → It looks like ∠C came from a larger circle than ∠D did.

T: Let's measure to find out if the angles turn the same number of degrees.

T: (Distribute and display a 180° protractor.)   What do you notice about this protractor?

S: It's half a protractor. → It's only a piece of a circular protractor. → It's got a straight edge.

T: Just like you measured angles with a circular protractor, you can measure angles with this 180° protractor. Protractors sometimes have two sets of numbers. We determine which number to read based off the side of the angle that touches zero. (Show a 40-degree angle as pictured to the right, aligning both sides to zero and discussing

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which set of numbers to read.)

T: (Model. Place the middle notch on the vertex of the angle. Line up a side with the zero or base line on the protractor. Read the number the second side length touches.)

T: With your partner, measure $\angle C$.

S: 60 degrees! No wait, 120 degrees. → It can't measure 120 degrees. It's an acute angle. 60 degrees. → Remember, we count up from the side of the angle at zero, so we are using the outside numbers for this angle.

T: Measure $\angle D$.

S: 60 degrees!

T: What did you discover? Discuss with your partner.

S: The arc lengths are different, but the degrees are the same. → Both angles are 60 degrees, but $\angle D$ looks different because the sides of the angle are shorter.

T: What would happen if we placed the angles on top of each other? Turn and talk. (Allow time for brief discussion.) Let's try! (Model.)

S: They match up! → The angles are the same size!

T: Imagine a circle drawn with the vertex of $\angle D$ as its center point, the end of one segment being the length to the arc and another circle drawn in the same way around $\angle C$.

T: What could you say about the two circles?

S: The circles would be different sizes. → The lengths of the sides of $\angle C$ would make a larger circle than the sides of $\angle D$. → The arcs and sides will be different lengths, but the angle will measure the same because each angle represents a fraction of 360 degrees.

Problem 3: Use multiple protractors to measure the same angle.

T: Look at the different protractors in front of you. What do you notice about them?

S: Some are 360° protractors and some are 180° protractors. → Some have just one set of numbers; others have two sets. → They are all different sizes. → The base line on this one is on the bottom of the protractor, but the base line on this one is above the plastic.

T: Line up your protractors using the center point, just like we did with our two circles at the beginning of the lesson. Do you see how these different protractors have different arcs?

S: Yes, some are small, and some are big.

T: Yes, but they all measure 360 degrees of a circle.

S: But some only measure 180 degrees.

T: That's because it is representing half a circle. Notice the tick marks on all of the different protractors.

S: Some are really close together!

T: Why is that?

S: It's on the smallest protractor, so that means the arc length is shorter than those of the other protractors.

T: Let's use at least three different protractors to measure $\angle E$.

Allow time for students to measure individually, in partners or in small groups, depending on the variety of

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protractors available in the classroom.

S: All three protractors showed this is a 120° angle!

T: What does that tell you about the side lengths of an angle?

S: The side lengths can be any length. → No matter where you measure on the circle, the number of degrees will always be the same. → We aren't measuring the sides of angles. The different sizes of protractors pick a different point on each segment where a circle could be and measures that.

T: Let's look at Problem 1(a) of the Problem Set together. Measure the angle that is shown.

S: I can't measure that angle. The image is too small! → I know what to do! We can make the segments of the angle longer. We just found out that the angle measure stays the same no matter what the side length is.

T: Use your straightedge to extend the sides of the angle until they are long enough for you to use the protractor to measure the angle. (Model.)

S: Now I can measure the angle!

> **NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:**
>
> Students who experience frustration with manipulating and reading a protractor may find success with virtual protractors, such as those found at the following website: http://www.teacherled.com/resources/anglemeasure/angleteach.swf
>
> Virtual protractors may be a viable option for classrooms that do not have a wide range or great number of protractors.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted time frame. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first.

Student Debrief (6 minutes)

Lesson Objective: Use varied protractors to distinguish angle measure from length measurement.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

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You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1 which angle had the same measure as ∠G? ∠I?

- In Problem 1 which angles had the same angle measures but different side length measures?

- Discuss your experience of measuring with different protractors (Problem 2).

- How many degrees did the angles in Problem 3 measure? What type of angle is Part (a)? We know a straight angle forms a straight line. Points A, B, and C create ∠ABC and $\overline{ABC}$. When three or more points are found on a line, we call them **collinear** points. Are points D, E, and F collinear? Why not?

- Take a look at your 180° protractor. Find pairs of numbers that label the two scales, such as 150° and 30°. Name another pairs of numbers. What do you notice about the pairs of numbers?

- How did the Application Problem help you to understand angle measure remains constant and is not a length measure?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

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C

D

E

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Name _____   Date _____

1. Use a protractor to measure the angles and then record the measurements in degrees.

   a.   b.
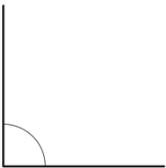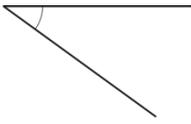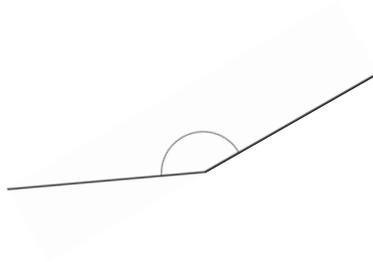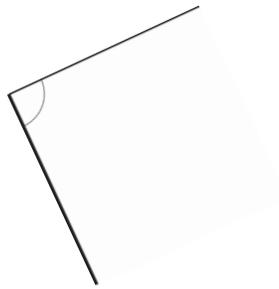
   c.   d.

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e.

f.

g.

h.

i.

j.

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2.

    a. Use three different-size protractors to measure the angle. Extend the lines as needed using a straightedge.

        Protractor #1: _____°

        Protractor #2: _____°

        Protractor #3: _____°
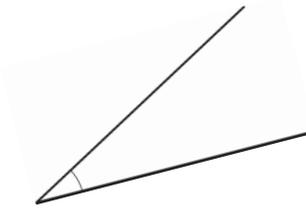
    b. What do you notice about the measurement of the above angle using each of the protractors?

3. Use a protractor to measure each angle. Extend the length of the lines if you need to. When you extend the lines, does the angle measure stay the same? Explain how you know.

    a.

C

B

A

F

E

    b.

D
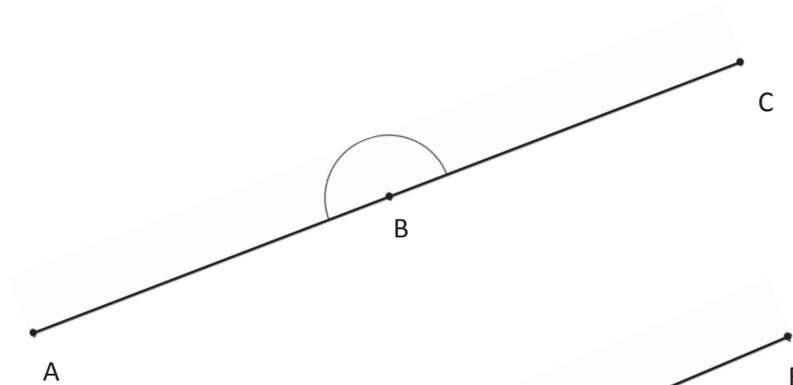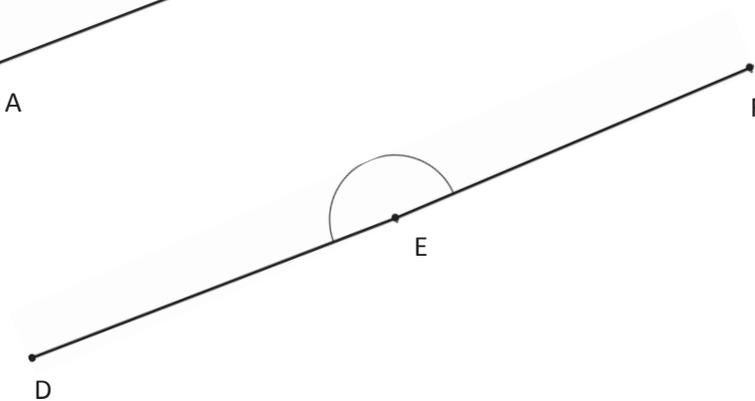
Name _____   Date _____

1. Use any protractor to measure the angles and then record the measurements in degrees.
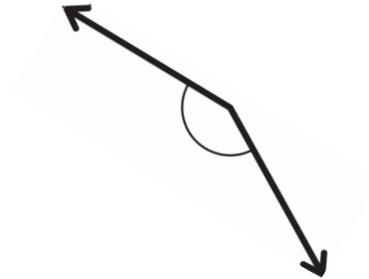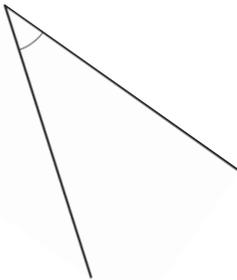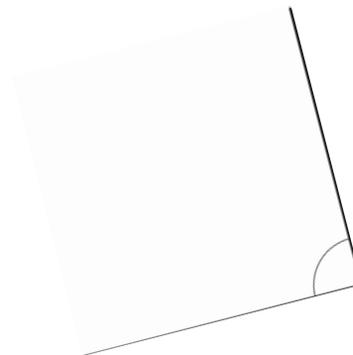
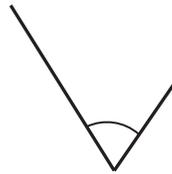   a.

    b.

    c.

    d.

Name _____   Date _____

1. Use a protractor to measure the angles and then record the measurements in degrees.
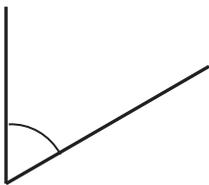
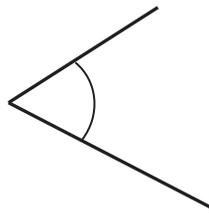   a.   b.
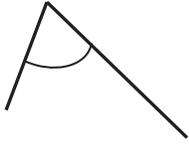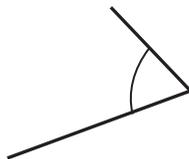
    c.   d.

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e.

f.

g.

h.

i.

j.

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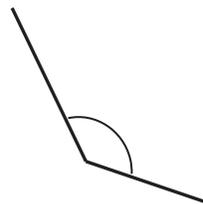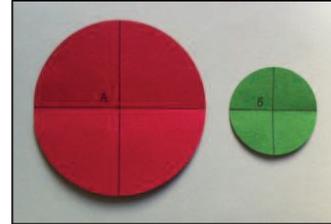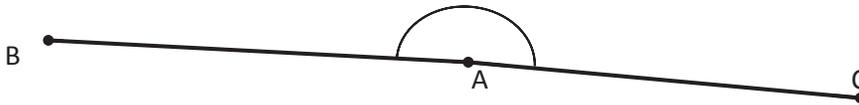
2.

  a.　Using the green and red circle cutouts from today's lesson, explain to someone at home how the cutouts can be used to show that the angle measures are the same even though the circles are different sizes. Write words to explain what you told him/her.

3.　Use a protractor to measure each angle. Extend the length of the lines if you need to. When you extend the lines, does the angle measure stay the same? Explain how you know.

    a.

B

A　　C

F

D

  b.

E

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