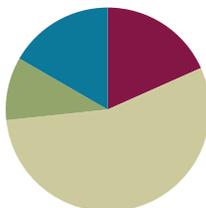


Lesson 5

Objective: Use a circular protractor to understand a 1-degree angle as $1/360$ of a turn. Explore benchmark angles using the protractor.

Suggested Lesson Structure

■ Fluency Practice	(11 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (11 minutes)

- Divide Using the Standard Algorithm **4.NBT.6** (3 minutes)
- Identify Two-Dimensional Figures **4.G.1** (4 minutes)
- Physiometry **4.G.1** (4 minutes)

Divide Using the Standard Algorithm (3 minutes)

Materials: (S) Personal white boards

Note: This reviews G4–M3–Lesson 16 content.

T: (Write $48 \div 2$.) On your boards, solve the division problem using the vertical method.

Continue with the following possible sequence: $49 \div 2$, $69 \div 3$, $65 \div 3$, $55 \div 5$, $58 \div 5$, $88 \div 4$, and $86 \div 4$.

Identify Two-Dimensional Figures (4 minutes)

Materials: (S) Personal white board, rulers

Note: This fluency reviews terms learned in G4–M4–Lessons 1–4.

T: (Project \overleftrightarrow{AB} . Point to the A.) Say the term for what I’m pointing to?

S: Point A.

T: (Point to the B.) Say the term.

S: Point B.

T: (Point to \overleftrightarrow{AB} .) Say the term.

S: Line AB.

T: Use your ruler to draw \overleftrightarrow{CD} on your personal boards.

S: (Draw a line with points C and D on the line.)

Continue with the following possible sequence: \overleftrightarrow{EF} , \overleftrightarrow{GH} , and obtuse $\angle IKJ$, acute $\angle LNM$, and right $\angle OQP$.

T: What's the relationship between \overleftrightarrow{QO} and \overleftrightarrow{QP} ?

S: The line segments are perpendicular.

T: Draw vertical line segment \overline{RS} .

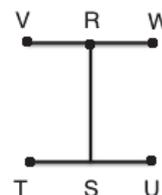
S: (Draw \overline{RS} .)

T: Draw \overline{TU} that is perpendicular to \overline{RS} .

S: (Draw \overline{TU} .)

T: Draw \overline{VW} that is perpendicular to \overline{RS} and parallel to \overline{TU} .

S: (Draw \overline{VW} .)



Physiometry (4 minutes)

Materials: (S) Personal white boards

Note: Kinesthetic memory is strong memory. This fluency reviews terms from G4–M4–Lessons 1–4.

T: Stand up.

S: (Stand up.)

T: Show me a point.

S: (Clench one hand in a fist and extend arm forward.)

T: Show me a line.

S: (Extend arms straight so that they are parallel with the floor. Open both hands.)

T: Show me a ray.

S: (Extend arms straight so that they are parallel with the floor. Clench one hand in a fist and leave the other hand open.)

T: Show me a ray pointing in the other direction.

S: (Open clenched hand and clench open hand.)

T: Show me a line segment.

S: (Extend arms straight so that they are parallel with the floor. Clench both hands into fists.)

T: Show me a right angle.

S: (Stretch one arm up directly at the ceiling. Stretch another arm directly towards a wall, parallel to the floor.)

T: Show me a different right angle.

S: (Stretch the arm pointing towards a wall directly up towards the ceiling. Move the arm pointing towards the ceiling so that it points directly towards the opposite wall.)

T: Show me an obtuse angle.

S: (Make an obtuse angle with arms.)

T: Show me an acute angle.

S: (Make an acute angle with arms.)

Continue with the following possible sequence: point, right angle, line segment, acute angle, line, right angle, and obtuse angle.

T: (Stretch one arm up directly at the ceiling. Stretch another arm directly towards a wall, parallel to the floor.) What type of angle am I making?

S: Right angle.

T: What is the relationship of the lines formed by my arms?

S: Perpendicular lines.

T: (Point to the classroom’s back wall.) Point to the walls that run perpendicular to the wall I’m pointing to.

S: (Point to the side walls.)

T: (Point to the front wall.)

S: (Point to the side walls.)

Continue pointing to one side wall, the back wall, the other side wall, and the front wall.

T: (Point to the back wall.) Point to the wall that runs parallel to the wall I’m pointing to.

S: (Point to the front wall.)

Continue pointing to one side wall, the front wall, and the other side wall.

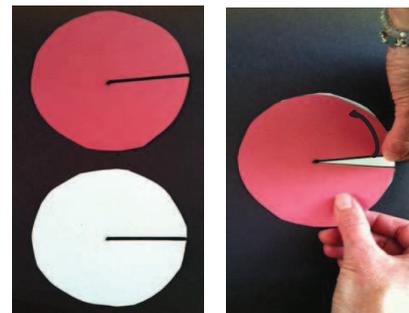
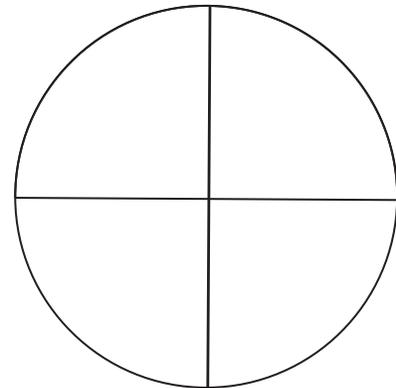
Application Problem (6 minutes)

Materials: (S) 1 paper circle from the Concept Development

Place right angle templates on top of the circle to determine how many right angles can fit around the center point of the circle. If necessary, team up with other students to share templates. (Overlaps are not allowed.)

How many right angles can fit?

Note: This Application Problem bridges from G4–M4–Lesson 2. Students will use the right angle templates that they made in class in order to build understanding as they measure right angles around the center point of a circle.



Concept Development (33 minutes)

Materials: (T) 2 paper circles with 5” diameter (same size, one red, one white, with a radius delineated by a strong, straight black segment) (S) 2 paper circles with 5” diameter (same size, one red, one white, with a radius cut into each one), circular

protractor with 4" diameter on paper or transparency

Directions for Constructing a Paper Protractor:

- Label and cut a radius into one red and one white paper circle.
- Stack the red circle on top of the white circle with the radii aligned and parallel to the desk's edge.
- Pinch the corner of the white circle directly below the slit, as shown above.
- To illustrate an angle, turn the segment given by the edge of the white region counterclockwise.

Problem 1: Reason about the number of turns necessary to make a full turn with different fractions of a full turn.

- T: What do you see as you turn this segment to the left?
 S: The white part is getting larger. The red part is getting smaller.
 T: Do you see an angle?
 S: Yes.
 T: Let's agree that the white region is the interior of the angle we are focusing on.
 T: (Demonstrate a quarter-turn.) Now show a quarter-turn of the segment to the left. (Expect some confusion.)
 S: (Show.)
 T: Make another quarter-turn of the segment to the left. What fraction of the circular region is white now?
 S: One half. → Two fourths.

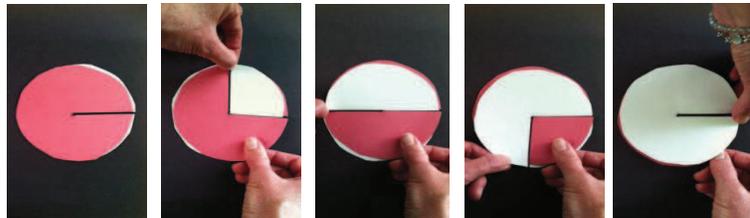
Continue the same process until the 360-degree turn is complete.

T: (List the following fractions on the board.)

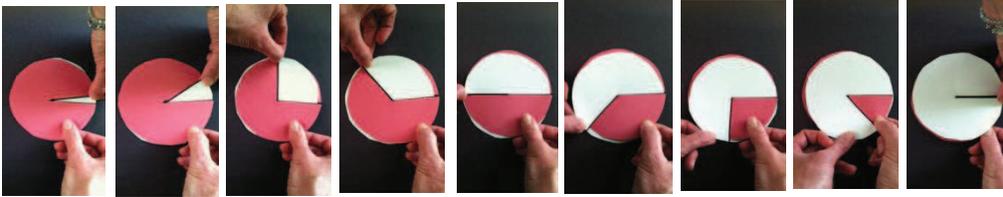
$$\frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad \frac{4}{4}$$

T: (Point to each fraction as you speak, pausing as students manipulate the turns.) Show $\frac{1}{4}$ turn, $\frac{2}{4}$ turn, now a $\frac{3}{4}$ turn, a $\frac{4}{4}$ turn. Is the angle getting larger or smaller?

- S: Larger!
 T: How many fourth-turns did it take to make one full turn?
 S: Four.



T: Now I want to break up a turn into eight equal parts. Count eighths with me.



T: Will one eighth-turn be less than or greater than one fourth-turn?

S: Less than.

T: One fourth-turn is the same as two eighth-turns (point to the listed fractions). Show me what you think would be one eighth-turn.

Repeat the same process of pointing to each eighth in order as the students open the angle.

T: Did it take more fourth-turns or eighth-turns to get all the way around?

S: Eighths.

T: How many eighth-turns did it take to make a whole turn?

S: Eight!

T: How many $\frac{1}{100}$ turns would it take to make a whole turn?

S: 100!

T: Would $\frac{1}{360}$ turn be smaller or larger than $\frac{1}{100}$ turn?

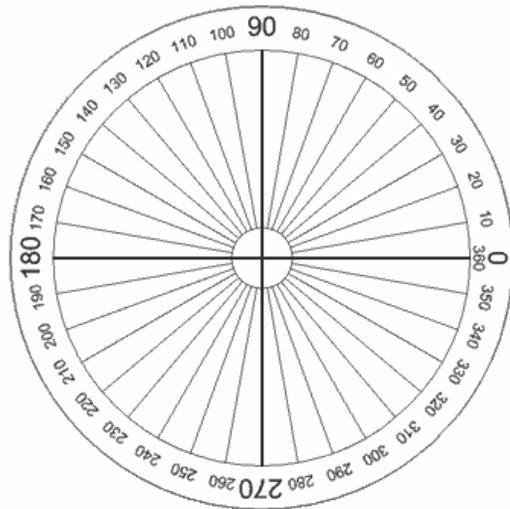
S: Smaller.

T: We have a special name for $\frac{1}{360}$ of a whole turn. It is called a **degree**! How many degrees are in one whole turn?

S: 360.

T: Yes!

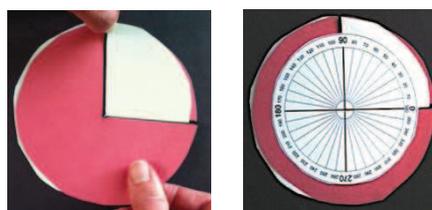
T: Here is a tool that has been partitioned and marked off to show 360 degrees. It is called a **protractor**. Take a moment to analyze it with your partner. What do you notice?



- S: It is shaped like a circle. → It is counting by tens starting at zero and going to the left. → It has 360 degrees. → It has bigger numbers at 90, 180, 270, and 360. → It has perpendicular lines. → There are lots of lines that could make angles if that center circle wasn't there in the middle covering the endpoints. → I see four right angles!
- T: Run your finger across your protractor from zero to the center point where the bold perpendicular lines cross. Let's call that the zero line, or base line, of our protractor because it will be the starting point from where we measure angles.

Problem 2: Use a circular protractor to determine that a quarter-turn or a right angle measures 90 degrees, a half turn measures 180 degrees, a three quarter-turn measures 270 degrees, and a full rotation measures 360 degrees.

- T: Show me a quarter-turn with your circles. Keep the base segment of your angle parallel to your desk.
- T: Put the zero line, or base line, on top of the bottom segment of your angle. Align the center point of the protractor with the vertex of the angle to the best of your ability.
- T: Adjust the circle's angle to match your right angle template. (Pause.) Remove the template and place the protractor to measure that angle. What do you notice?
- S: The quarter-turn matches the bold lines of the protractor. → It's 90 degrees! → One fourth-turn is 90 degrees. → A right angle measures 90 degrees.
- T: Do a half-turn and see how many degrees your angle is?
- S: 180 degrees.
- T: Turn another quarter- or fourth-turn.
- S: 270 degrees.
- T: And one last quarter- or fourth-turn?
- S: 360 degrees. → Zero degrees.
- T: What does your angle look like right now?
- S: It's all white.
- T: A zero-degree angle is when we have not turned at all. We have made one full turn of 360 degrees. There are 360 degrees in a full turn.
- T: How many 90-degree angles or right angles are there in a full turn?
- S: Four right angles.
- T: How do you know?
- S: Because we made four quarter-turns and each one was 90 degrees. → It is easy to see them because of the bold perpendicular lines on the protractor.



NOTES ON PROTRACTORS:

Circular protractors come in many different sizes and formats. Many include not one but two numbered scales. Since the lesson's objective is to understand a one-degree angle, we suggest using the template which only has one set of angle measures moving counter-clockwise counting by tens. Its simplicity may well ground students' initial understanding so that when they encounter a *regular* protractor in the next lesson, they will have internalized the meaning of a degree and the measures of acute and obtuse angles.

- T: Using your white circle, position your protractor with the zero or base line on top of the black segment, matching up the center point of the circle with the center point of the protractor.
- T: Estimate to make a point at 90 degrees. Draw a line segment from the center point to that point. What have you drawn?
- S: A right angle. → A 90-degree angle. → Perpendicular lines.
- T: Now make a point at 45 degrees. Draw a line segment from the center point to the point you just made. What have you made?
- S: A 45-degree angle.
- T: Yes! What do you notice?
- S: The 45-degree angle is half as big as the 90-degree angle. → Two forty-five degree angles are the same as one 90-degree angle. → $2 \times 45 = 90$.



**NOTES ON
MULTIPLE MEANS OF
ACTION AND
EXPRESSION:**

Ease the task of drawing benchmark angles for students with the following suggestions:

- Provide larger paper circles of thicker cardstock that may be easier to manipulate.
- Provide circles with pre-drawn markings.
- Provide the complete list of benchmark angles, liberating students to focus on reading and drawing angles with a protractor.

Problem 3: Measure and draw benchmark angles with the protractor.

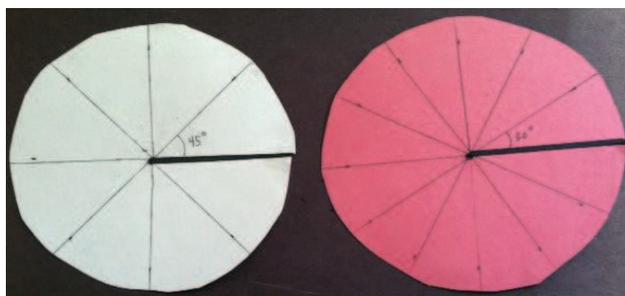
- T: Now let's work to measure and draw benchmark angles using your circles and protractors.
- T: We have already started Set A, using your white circle. Continue turning your circle, aligning the zero or base line with each last segment drawn. Be sure to keep your protractor's center point on the center point of the circle. Draw new 45-degree angles until you have gone a whole turn. Let me demonstrate. (Demonstrate silently.)
- T: Draw Set B on your red circle just as you did Set A, but this time draw 30-degree angles. This full turn will be made of 30-degree angles. Draw 30-degree angles until you have made a whole turn.

Set A

45 degrees

Set B

30 degrees



- T: Place the center point of the protractor on the shared endpoints of the segments on your white circle. Align the zero line with the black segment. What are the measurements of the angles you have drawn?
- S: 0 degrees, 45 degrees, 90 degrees, 135 degrees, 180 degrees, etc.

T: Trace each angle separately with your finger moving from the smallest angle to the largest.

Repeat the process with the sequence of 30-degree angles.

T: All of these are benchmark angles. Let's use our Problem Set to further explore them.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set. Some students might be allowed to complete the drawing of the benchmark angles while others start into the Problem Set. Take 10 minutes for the Problem Set as always, with the understanding that the variation in work completed may vary considerably.

Student Debrief (10 minutes)

Lesson Objective: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the questions below to lead the discussion.

- When you listed the benchmark angles, did you notice any numerical patterns?
- You listed some measures of acute and obtuse angles. What would be some measurements of other acute angles? Obtuse angles?
- A full turn is 360 degrees. What could you do to find the **degree** measure of an angle that takes 10 turns to make a whole turn?
- How did you respond to the final question?

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 5 Problem Set 4•4

Name: Jack Date: _____

1. Make a list of the measures of the benchmark angles you drew starting with Set A. Round each angle measure to the nearest 5 degrees. Both sets are started for you.

a. Set A: 45 degrees, 90 degrees, 135 degrees, 180 degrees, 225 degrees, 270 degrees, 315 degrees, 360 degrees

b. Set B: 30 degrees, 60 degrees, 90 degrees, 120 degrees, 150 degrees, 180 degrees, 210 degrees, 240 degrees, 270 degrees, 300 degrees, 330 degrees, 360 degrees

2. Circle any angle measures that appear on both lists. What do you notice about them?

They are all quarter turns and right angles.

3. List the angle measures from Problem 1 that are acute. Trace each angle with your finger as you say its measurement.

30 degrees, 45 degrees, 60 degrees

4. List the angle measures from Problem 1 that are obtuse. Trace each angle with your finger as you say its measurement.

120 degrees, 135 degrees, 150 degrees

COMMON CORE Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor. engage^{ny} 4.B.9
Date: 9/6/13

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 5 Problem Set 4•4

5. We found out today that 1 degree is $\frac{1}{360}$ of a whole turn. It is 1 out of 360 degrees. That means a 2 degree angle is $\frac{2}{360}$ of a whole turn. What fraction of a whole turn is each of the benchmark angles you listed in Problem 1?

$\frac{30}{360}$, $\frac{45}{360}$, $\frac{60}{360}$, $\frac{90}{360}$, $\frac{120}{360}$, $\frac{135}{360}$, $\frac{150}{360}$, $\frac{180}{360}$, $\frac{210}{360}$, $\frac{225}{360}$, $\frac{240}{360}$, $\frac{270}{360}$, $\frac{300}{360}$, $\frac{315}{360}$, $\frac{330}{360}$, $\frac{360}{360}$

6. How many 45 degree angles does it take to make a full turn?

It takes 8 45 degree angles to make a full turn.

7. How many 30 degree angles does it take to make a full turn?

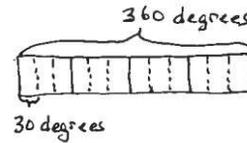
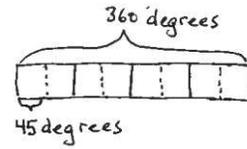
It takes 12 30 degree angles to make a full turn.

8. If you didn't have a protractor, how could you reconstruct the quarter of it from 0 degrees to 90 degrees?

You could use two 45 degree angles or three 30 degree angles put together.

COMMON CORE Lesson 5: Use a circular protractor to understand a 1-degree angle as $\frac{1}{360}$ of a turn. Explore benchmark angles using the protractor. engage^{ny} 4.B.10
Date: 9/6/13

- If you were to draw a tape diagram to represent one whole-turn and the benchmark angles of Set A, what would you do? Set B?
- Shade in the region of a 45-degree angle on your white circle. What fraction of the whole turn is that? Do the same for your 30-degree angle.
- What if you shaded in a region defined by a 120-degree angle on your red circle? What fraction of the whole circle is that?
- Use your protractor to explain to your partner what a degree is.
- Use your protractor to trace some benchmark angles.
- If you didn't have a protractor, how could you construct one?



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

5. We found out today that 1 degree is $\frac{1}{360}$ of a whole turn. It is 1 out of 360 degrees. That means a 2-degree angle is $\frac{2}{360}$ of a whole turn. What fraction of a whole turn is each of the benchmark angles you listed in Problem 1?
6. How many 45-degree angles does it take to make a full turn?
7. How many 30-degree angles does it take to make a full turn?
8. If you didn't have a protractor, how could you reconstruct the quarter of it from 0 degrees to 90 degrees?

Name _____

Date _____

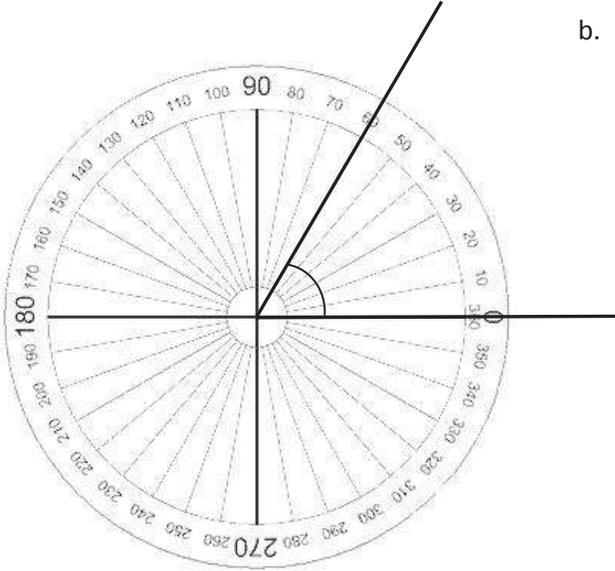
1. How many right angles make a full turn?
2. What is the measurement of a right angle?
3. What fraction of a full turn is 1 degree?
4. Name at least four benchmark angle measurements.

Name _____

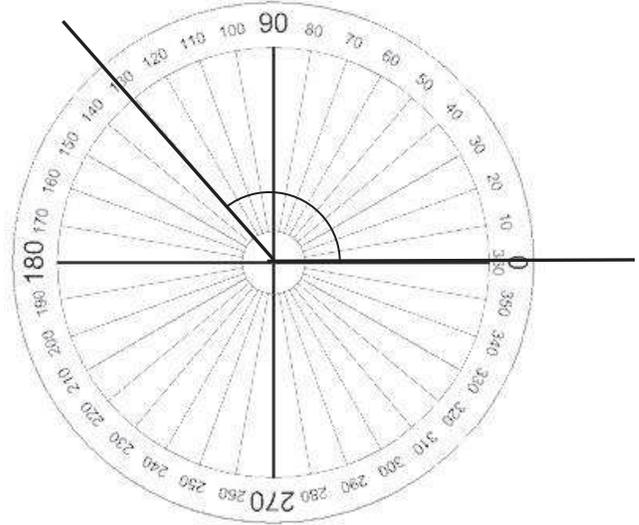
Date _____

1. Identify the measures of the following angles.

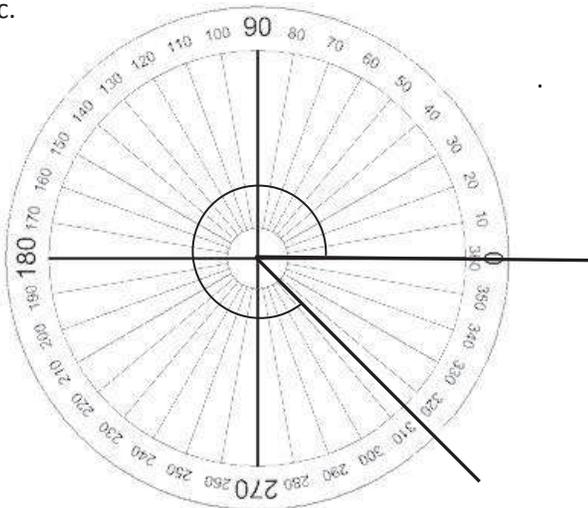
a.



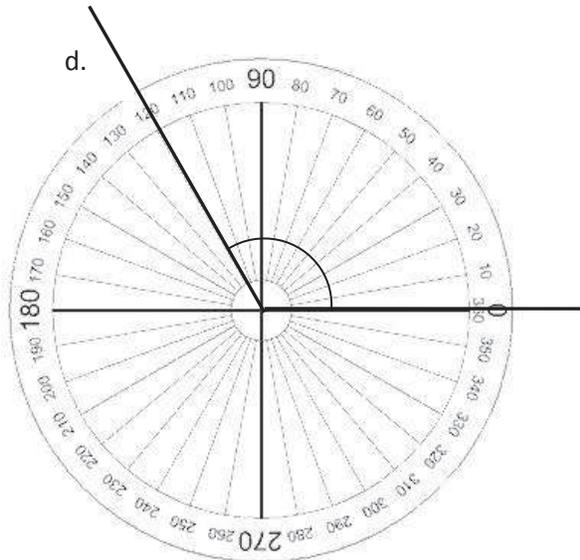
b.



c.



d.



2. If you didn't have a protractor, how could you construct one? Use words, pictures, and numbers to explain in the space below.