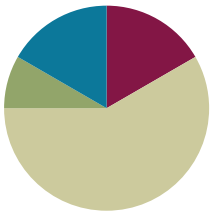
Lesson 13

Objective: Analyze and classify triangles based on side length, angle measure or both.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Application Problem	(5 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (10 minutes)

- Divide Three Different Ways **4.NBT.6** (5 minutes)
- Physiometry **4.G.3** (3 minutes)
- Lines of Symmetry **4.G.3** (2 minutes)

Divide Three Different Ways (5 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews concepts covered in G4–Module 3. Alternately, have students choose to solve the division problem using just one of the three methods.

T: (Write 532 ÷ 4.) Solve this problem by drawing number disks.
S: (Solve with number disks.)
T: Solve 532 ÷ 4 using the area model.
S: (Solve with the area model.)
T: Solve 532 ÷ 4 using the standard algorithm.
S: (Solve with the standard algorithm.)

Continue with this possible suggestion: 854 ÷ 3.

Physiometry (3 minutes)

Materials: (S) Personal white boards

Note: Kinesthetic memory is strong memory. This fluency reviews terms learned in G4–M4–Lesson 12.

T: Stand up.

T: Am I trying to make my body position look symmetrical?

T: (Raise left arm so that fingers are pointing directly to the wall. Leave the other arm hanging down.) Is my position symmetrical now?

S: No.

Continue with other symmetrical and non-symmetrical positions.

T: With your arms, model a line that runs parallel to the floor. Are you modeling a symmetrical position?

S: Yes.

T: Model a right angle. Are you modeling a symmetrical position?

S: No.

T: Model a line segment that runs parallel to the floor. Are you modeling a symmetrical position?

S: Yes.

Lines of Symmetry (2 minutes)

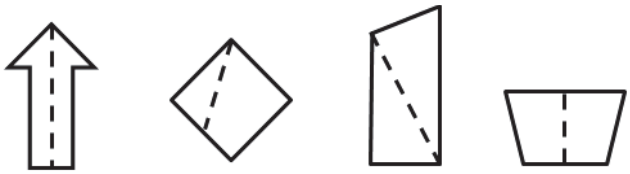
Note: This fluency reviews G4–M4–Lesson 12.

T: (Project arrow with a line of symmetry. Point to the line of symmetry.) Is this a line of symmetry?

S: Yes.

T: (Project the diamond. Point to the non-symmetrical line.) Is this a line of symmetry?

S: No.

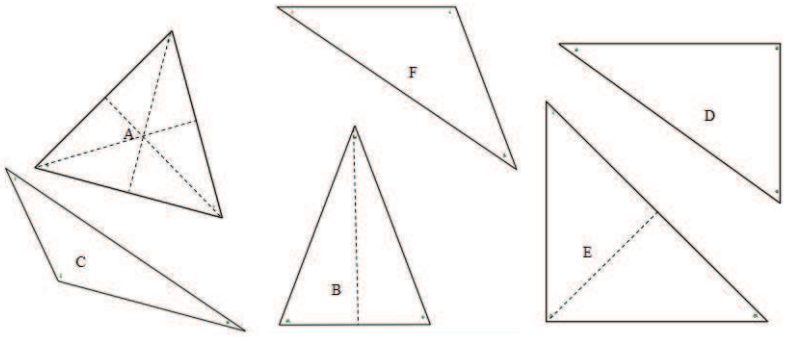


Continue process for the remaining graphics.

Application Problem (5 minutes)

Fold Triangles A, B, and C to show their lines of symmetry. Use a straightedge to trace each fold. Discuss with your partners the relationships of symmetric shapes to angles and side lengths.

Note: This Application Problem connects the objective in G4–M–Lesson 12 for lines of symmetry to discovering the attributes of triangles in today's lesson. Prepare the triangles ahead of time by cutting them out from the activity template. Each student or partner group should have their own copy.



Concept Development (35 minutes)

Materials: (T) Set of Triangles A–C and A–F, Practice Sheet, graph paper, ruler (S) Set of Triangles A–C and A–F per group, Practice Sheet, ruler, protractor, graph paper

Problem 1: Discover the attributes of various triangles.

T: What types of attributes can triangles have?

S: Well, they must have three sides, so they also have three angles. → But their sides can be different. Some are short and some are long, or sometimes they are the same length. → Yeah, triangles can also have the same or different types of angles, like acute, obtuse, or right. → And some have lines of symmetry, and others don't.

T: Think about the types of angles and the lengths of the sides of triangles as we complete this activity.

Separate students into small groups of three students each. Provide each group with one of each triangle on the activity template, Triangles A–F. Instruct students to investigate the given triangle cutouts using rulers and protractors. Students should record their findings in the Attributes column of the Practice Sheet, including measures of sides and angles, as well as other general observations. It may be helpful for students to also record the angle and side length measurements on the cutouts as well. Students should quickly sketch each triangle in the first column. Allow students six to eight minutes for this activity.

T: Now, take a moment with your group to compare your findings. Discuss ways in which some triangles might be classified into different groups.

Students discuss.

Problem 2: Classify triangles by side length and angle measure.

T: Tell me how you sorted your triangles by side length.

S: Triangles B, E, and F each had two sides that were the same length. → Triangles C and D had sides that all measured different lengths. → Triangle A is the only triangle that has three sides that are all the same length!

T: Let's record your findings. You just classified some triangles by the length of their sides. Let's label the first of the classification columns as *Side Length*.

MP.6

T: There are three kinds of triangles you discovered. **Equilateral triangles**, like Triangle A, have all sides that are equal in length.

S: That's easy to remember because *equilateral* starts with the same sound as the word *equal*.

T: **Isosceles triangles** are like Triangles B, E, and F. They have at least two sides with the same length.

T: Triangles C and D are classified as **scalene triangles**. None of their side lengths are the same.

T: To show that certain sides are the same length, we draw a tick mark on each same length segment.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Remembering the names that classify triangles may present a challenge for English language learners and others. Present helpful mnemonic devices. The word *isosceles*, for example, starts with the sound *eyes*. We have *two* eyes; similarly, an isosceles triangle has at least *two* equal sides. Encourage students to come up with their own way to remember and then to share with others.

(Draw a tick mark on each side.) It's your turn. What other triangles need tick marks?

S: Triangles B, E, and F need just two tick marks.

T: Why don't Triangles C and D need tick marks?

S: All their sides have different lengths.

T: Tell me about how you sorted the triangles based on the angles you measured.

S: Triangles D and E had one right angle.

MP.6

S: Triangles C and F had one obtuse angle.

S: All of the triangles had acute angles. → Triangles A and B had only acute angles.

T: Label the second of the classification columns of your table *Angle Measure*. Record your findings. If a triangle had an obtuse angle, we classify those as **obtuse triangles**. If a triangle has one right angle, we call it a **right triangle**. What are triangles called that have only acute angles?

S: **Acute triangles**!

T: What angle symbol do we know to show the classification of right triangles?

S: The small square!

Problem 3: Determine presence of angles of specific measure in triangles.

T: Fold Triangle B on its line of symmetry. What do you notice about the two sides that line up?

S: They are the same length! That means we measured correctly. It is an isosceles triangle.

T: What about the two base angles that folded on top of each other?

S: The two angles are the same size! I wonder if that has something to do with the two sides being the same.

T: Let's check. Fold another isosceles triangle, Triangle E or F.

S: Those sides that fold together are the same and the angles are too!

T: Use those findings to make some conclusions about equilateral triangles. Fold Triangle A on each of its lines of symmetry.

S: No matter which symmetry line we folded, the sides were the same length and the angles matched up. → So, if all of the angles are lining up, doesn't that mean all of the angles have the same measure? → Yeah! And that means all the sides are the same length. And we knew that when we measured with our rulers and protractors. → Equilateral triangles are a lot like isosceles triangles.

T: An isosceles triangle has at least two sides that measure the same length. Do equilateral triangles have two sides that are the same length?

S: Yes. → Yes, but actually three.

T: An isosceles triangle has two angles with the same measure. Do equilateral triangles have two

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners and others may feel overwhelmed with the many new terms introduced in this lesson. Encourage students to record *isosceles, equilateral,* and *scalene* in their personal math dictionaries. Students may, for example, draw an example of each type of triangle and then define the triangles in their first language, if helpful. Create a classroom chart with examples for each type of triangle so that students may reference it during the Problem Set and further triangle work.

angles with the same measure?

S: Yes. → Yes, but actually three.

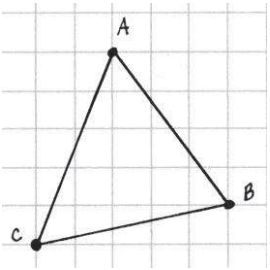
T: We can say that an equilateral triangle is a special isosceles triangle. It has everything an isosceles triangle has, but then it has a little more, like three sides and three angles with the same measure, not just two.

T: Triangle D has a right angle. Fold the other two angles into the right angle. (Demonstrate.) It's your turn.

S: Neat, the two other angles fit perfectly into the right angle.

T: What does that tell you about the measure of both of the other angles in a right triangle?

S: The other two angles add together to make 90 degrees.



Problem 4: Define triangle.

T: What do we know about triangles that will help us draw one?

S: Triangles have three sides and three angles. → We could draw three segments that meet together. → Those three segments will make the three angles. → When we learned about angles, we drew them by drawing two rays from one point.

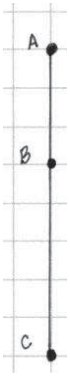
T: On graph paper, plot three points and label them *A, B*, and *C*. Connect those points with rays. What have you created?

S: A triangle!

T: (Plot three collinear points labeled *A, B*, and *C*.) What is the problem here?

S: They already are connected. You just drew a line. A, B, and C are all on one line.

T: Use your triangle to help you define the word triangle to your partner.



S: My triangle has three segments and three angles. → My triangle was formed from three points connected by three segments. → My triangle was formed from three points that were not in a line and connected by segments. → Two of my points can be in a line, but not all three.

T: Identify your triangle as ΔABC. (Write ΔABC.) Classify your triangle by side length and angle measure.

Students do so.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

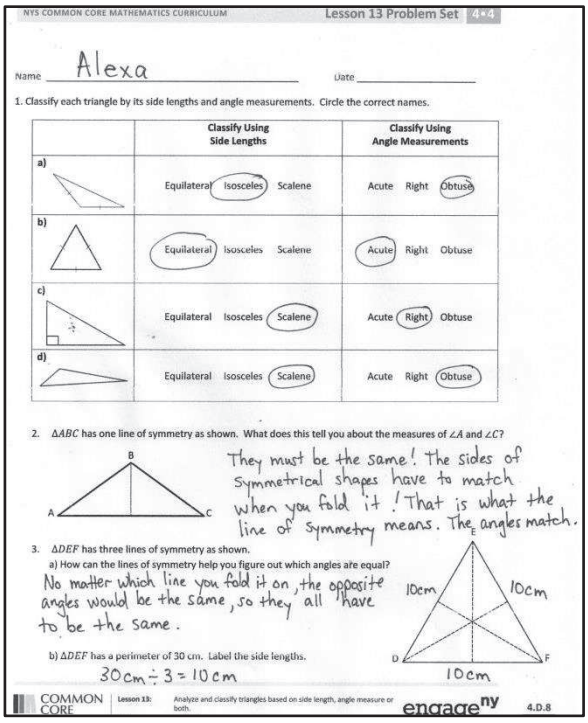
Lesson Objective: Analyze and classify triangles based on side length, angle measure, or both.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- How do the tick marks and angle symbols allow classification of triangles without using tools in Problem 1?
- What strategy did you use to solve Problem 3(b)?
- Explain your answer to Problem 5(b). The word *collinear* describes three points that are in a line.
- A triangle can be defined as three points that are not collinear and the line segments between them. Discuss this definition with your partner. Make sure you understand it completely.
- How many lines of symmetry can be found in **scalene triangles**? **Equilateral triangles**? **Isosceles triangles**?
- Can you determine whether or not a triangle will have a line of symmetry just by knowing whether it is an **acute triangle** or an **obtuse triangle**? How about scalene or isosceles? Sketch an example of a scalene and isosceles triangle to verify your answer.
- Sketch some examples to prove your answer to Problem 6. How many acute angles do **right triangles** have?
- How did the Application Problem connect to today's lesson?



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 13 Problem Set 4•4

Name Alexa Date

1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	Classify Using Angle Measurements
a)	Equilateral (Isosceles) Scalene	Acute Right (Obtuse)
b)	(Equilateral) Isosceles Scalene	(Acute) Right Obtuse
c)	Equilateral Isosceles (Scalene)	Acute (Right) Obtuse
d)	Equilateral Isosceles (Scalene)	Acute Right (Obtuse)

2. ΔABC has one line of symmetry as shown. What does this tell you about the measures of $\angle A$ and $\angle C$?

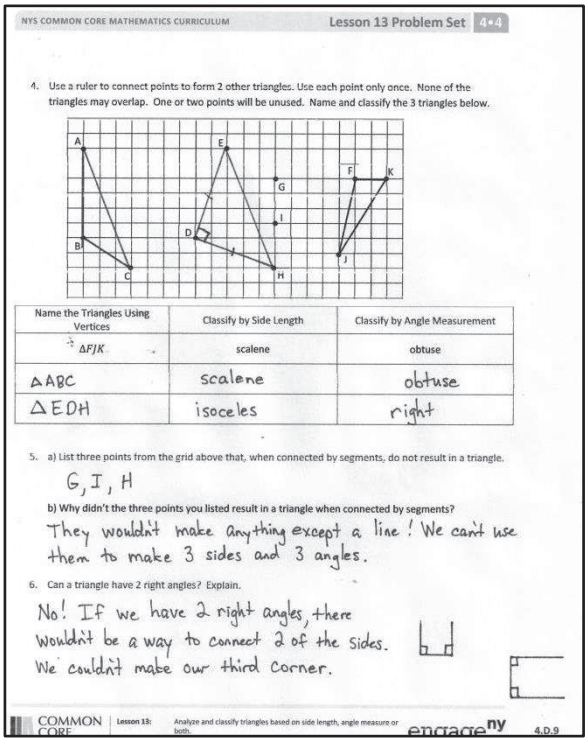
They must be the same! The sides of symmetrical shapes have to match when you fold it! That is what the line of symmetry means. The angles match.

3. ΔDEF has three lines of symmetry as shown.
a) How can the lines of symmetry help you figure out which angles are equal?
No matter which line you fold it on, the opposite angles would be the same, so they all have to be the same.

b) ΔDEF has a perimeter of 30 cm. Label the side lengths.
30cm ÷ 3 = 10cm

10cm 10cm 10cm

COMMON CORE Lesson 13: Analyze and classify triangles based on side length, angle measure or both. engage^ny 4.D.8



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 13 Problem Set 4•4

4. Use a ruler to connect points to form 2 other triangles. Use each point only once. None of the triangles may overlap. One or two points will be unused. Name and classify the 3 triangles below.

Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
ΔFJK	scalene	obtuse
ΔABC	scalene	obtuse
ΔEDH	isoceles	right

5. a) List three points from the grid above that, when connected by segments, do not result in a triangle.
G, I, H

b) Why didn't the three points you listed result in a triangle when connected by segments?
They wouldn't make anything except a line! We can't use them to make 3 sides and 3 angles.

6. Can a triangle have 2 right angles? Explain.
No! If we have 2 right angles, there wouldn't be a way to connect 2 of the sides. We couldn't make our third corner.

COMMON CORE Lesson 13: Analyze and classify triangles based on side length, angle measure or both. engage^ny 4.D.9

Exit Ticket (3 minutes)

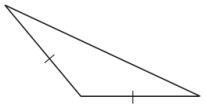
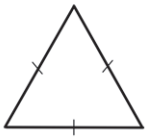
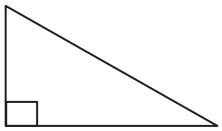
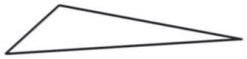
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____ Date _____

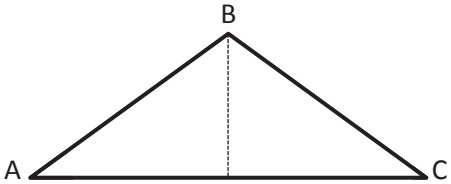
Sketch of Triangle	Attributes (Include side lengths, angle measures.)	Classification	
A			
B			
C			
D			
E			
F			

Name _____ Date _____

1. Classify each triangle by its side lengths and angle measurements. Circle the correct names.

	Classify Using Side Lengths	Classify Using Angle Measurements
a. 	Equilateral Isosceles Scalene	Acute Right Obtuse
b. 	Equilateral Isosceles Scalene	Acute Right Obtuse
c. 	Equilateral Isosceles Scalene	Acute Right Obtuse
d. 	Equilateral Isosceles Scalene	Acute Right Obtuse

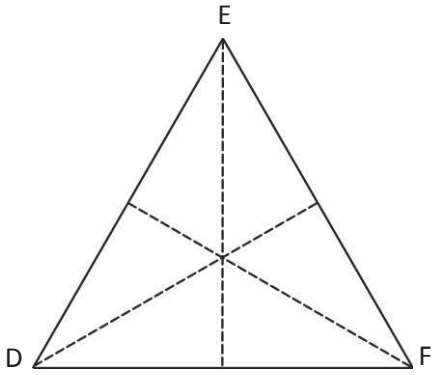
2. ΔABC has one line of symmetry as shown. What does this tell you about the measures of $\angle A$ and $\angle C$?



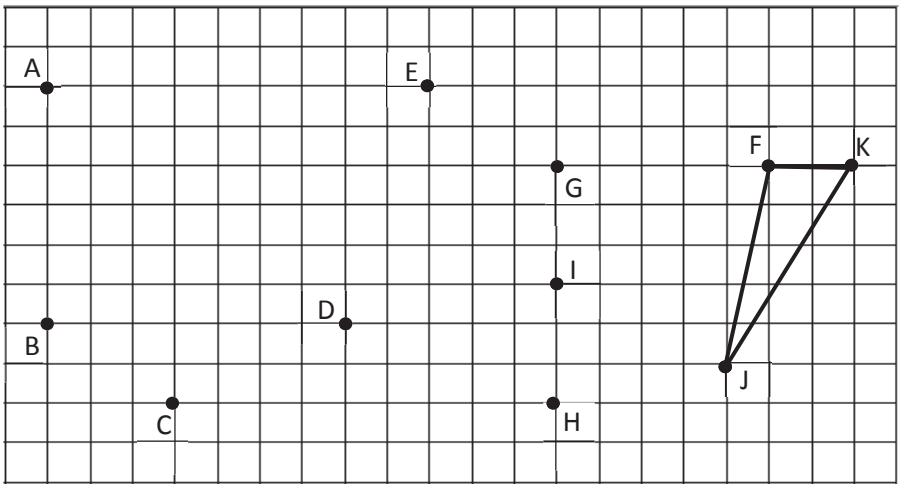
3. ΔDEF has three lines of symmetry as shown.

   a. How can the lines of symmetry help you figure out which angles are equal?

   b. ΔDEF has a perimeter of 30 cm. Label the side lengths.



4. Use a ruler to connect points to form 2 other triangles. Use each point only once. None of the triangles may overlap. One or two points will be unused. Name and classify the 3 triangles below.



Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
ΔFJK	scalene	obtuse

5.

a. List three points from the grid above that, when connected by segments, do not result in a triangle.

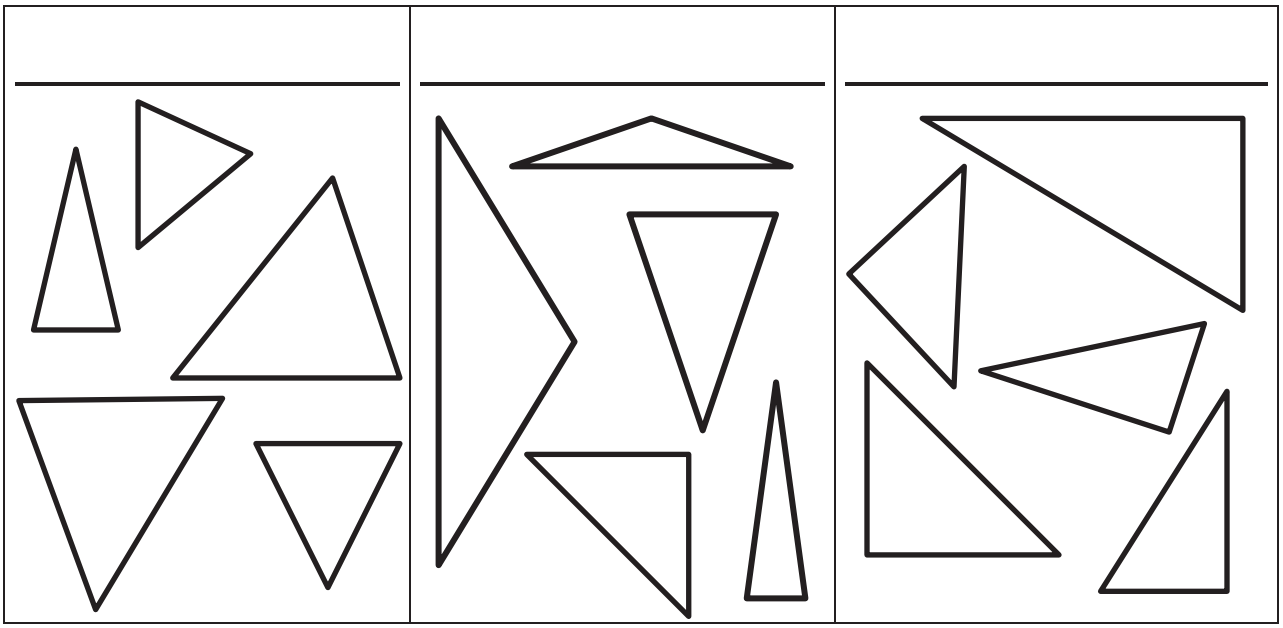
b. Why didn't the three points you listed result in a triangle when connected by segments?

c. Can a triangle have 2 right angles? Explain.

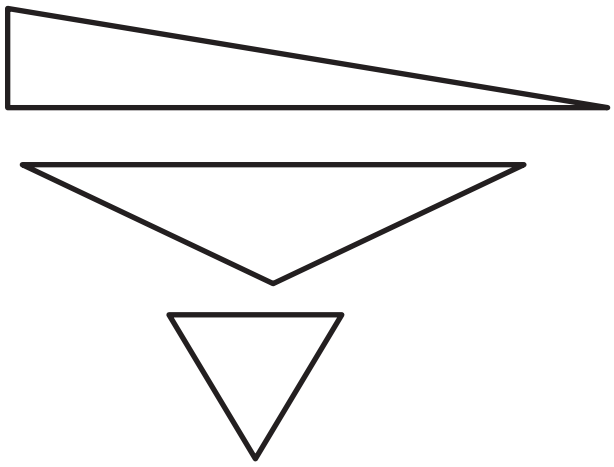
Name _____ Date _____

Use appropriate tools to solve the following problems.

1. The triangles below have been sorted by shared attributes (side length ***or*** angle type). Use the words *acute, right, obtuse, scalene, isosceles,* or *equilateral* to label the headings to identify the way the triangles have been sorted.



2. Draw a line to identify each triangle according to angle type *and* side length.



Acute

Obtuse

Right

Isosceles

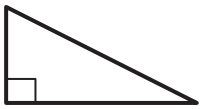
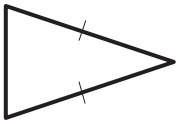
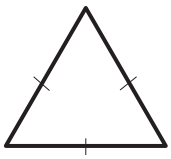
Equilateral

Scalene

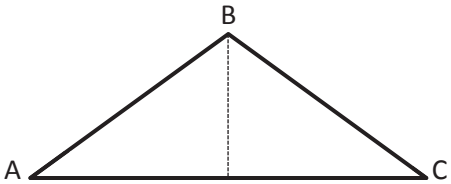
3. Identify and draw any lines of symmetry in the triangles in Problem 2.

Name _____ Date _____

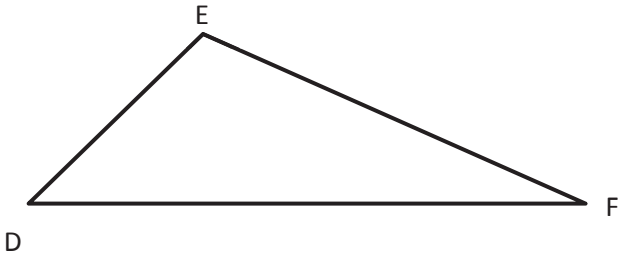
1. Classify each triangle by its side lengths and angle measurements. Circle the correct names. Use a ruler and a right angle template to prove your classifications.

	Classify Using Side Lengths	Classify Using Angle Measurements
a. 	Equilateral Isosceles Scalene	Acute Right Obtuse
b. 	Equilateral Isosceles Scalene	Acute Right Obtuse
c. 	Equilateral Isosceles Scalene	Acute Right Obtuse
d. 	Equilateral Isosceles Scalene	Acute Right Obtuse

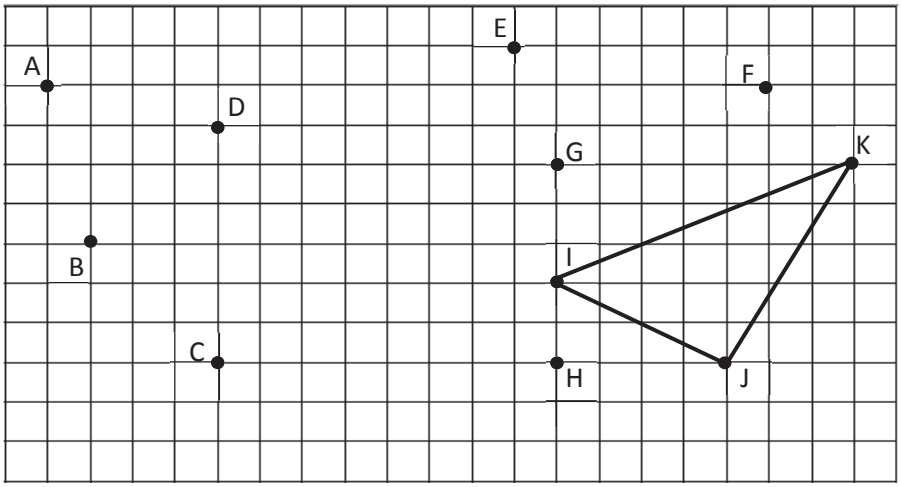
2. a. ΔABC has one line of symmetry as shown. Is the measure of $\angle A$ greater than, less than, or equal to $\angle C$?



b. ΔDEF is scalene. What do you observe about its angles? Explain.



3. Use a ruler to connect points to form two other triangles. Use each point only once. None of the triangles may overlap. Two points will be unused. Name and classify the three triangles below.



Name the Triangles Using Vertices	Classify by Side Length	Classify by Angle Measurement
ΔIJK		

4. If the perimeter of an equilateral triangle is 15 cm, what is the length of each side?

5. Can a triangle have more than one obtuse angle? Explain.

6. Can a triangle have one obtuse angle and one right angle? Explain.